RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2015-18]
B.A./B.Sc. SECOND SEMESTER (January – June) 2016
Mid-Semester Examination, March 2016

Date : 17/03/2016 **MATHEMATICS (Honours)**
Time : 11 am – 1 pm Paper : II Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

1. **Answer any four questions :** [4×3]

 a) Prove that $\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2+\left(c+\frac{1}{c}\right)^2 \geq 33\frac{1}{3}$, where a, b, c are all positive reals satisfying a+b+c = 1.

 b) Prove that $\left(1-\frac{1}{n+1}\right)^{n+1} > \left(1-\frac{1}{n}\right)^{n}$, where n is a positive integer.

 c) Prove that the least value of $x+2y+4z$ is $4\sqrt{3}$ where x,y,z are positive reals satisfying $x^2y^3z=8$.

 d) Find the sum of 99^{th} powers of the roots of the equation $x^7-1=0$.

 e) Expand $\cos^8\theta$ in a series of cosines of multiples of θ.

 f) Find the ratio of the principal values of $(1+i)^{1-i}$ and $(1-i)^{1+i}$.

2. **Answer any two questions :** [2×4]

 a) Discuss the convergence of the series $\sum_2^\infty \frac{1}{n(\log n)^p}, p>0$.

 b) Show that the series $\frac{1}{(1+a)^p}-\frac{1}{(2+a)^p}+\frac{1}{(3+a)^p}-\ldots, a>0$ is absolutely convergent if p > 1 and conditionally convergent if $0<p\leq 1$.

 c) Prove that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ converges to log 2 but the re-arranged series
 $1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+\ldots$ converges to $\frac{1}{2}\log 2$.

3. **Answer any two questions :** [2×2·5]

 a) Prove that a closed subset of a compact set is compact.

 b) Prove that if K be a non-empty compact set in $\mathbb{R}$ then K has a least element.

 c) Prove that H is not compact where $H=[0,\infty)$.

Group – B

4. **Answer any three questions :** [3×2]

 a) If AB = B and BA = A show that A and B are both idempotent matrices.

 b) Prove that $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = (a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$, where $\omega^3=1$.

c) If the system of equations

$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0$$

has non-zero solutions, prove that either $a + b + c = 0$ or $a = b = c$.

d) P is an n×n real orthogonal matrix with det P = –1. Prove that $P + I_n$ is a singular matrix.

e) A non-singular matrix P commutes with P^t. Prove that P^t commutes with P^{-1} and $P^{-1}P^t$ is orthogonal.

5. Obtain non-singular matrices P and Q such that PAQ is the fully reduced normal form of $A = \begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 4 & 0 \\ 3 & 1 & 6 & 3 \end{pmatrix}$. [4]

Answer <u>any one</u> question : [1×15]

6. a) Prove that if for a basic feasible solution x_B of a linear programming problem
Maximize z = cx
subject to $Ax = b$, $x \geq 0$
we have $z_j - c_j \geq 0$ for every column a_j of A, then x_B is an optimal solution. [6]

b) $x_1 = 1, x_2 = 1,\ x_3 = 1$ and $x_4 = 0$ is a feasible solution of the system of equations

$$x_1 + 2x_2 + 4x_3 + x_4 = 7$$
$$2x_1 - x_2 + 3x_3 - 2x_4 = 4$$

Reduce the F.S to two different B.F.S. [4]

c) Solve the following L.P.P by simplex method :

Maximize $z = 2x_1 + 3x_2 + x_3$

Subject to
$$-3x_1 + 2x_2 + 3x_3 = 8$$
$$-3x_1 + 4x_2 + 2x_3 = 7$$
$$x_1, x_2, x_3 \geq 0$$
[5]

7. a) Prove that every extreme point of the convex set of all feasible solutions of the system Ax = b, $x \geq 0$ corresponds to a basic feasible solution. [5]

b) Solve the L.P.P. by simplex method :

Maximize $z = 2x_1 + 5x_2$

subject to
$$x_1 + 2x_2 \leq 8$$
$$x_1 \leq 4$$
$$0 \leq x_2 \leq 3$$

and x_1 is unrestricted in sign. [6]

c) State Fundamental Theorem of L.P.P.
Prove that the set of all convex combinations of a finite number of points is a convex set. [1+3]

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